Name:	
Class:	

JAMES RUSE A H S

GESTA NON VERBA

Term 1 Task 2 2014

MATHEMATICS

General Instructions:

- Reading Time: 5 minutes.
- Working Time: 2 hours.
- Write in black pen.
- Board approved calculators & templates may be used
- A Standard Integral Sheet is provided.
- In Question 6 - 9, show all relevant mathematical reasoning and/or calculations.
- Marks may not be awarded for careless or badly arranged working.

Total Marks 85

Section I: **5 marks**

- Attempt Question 1 – 5.
- Answer on the Multiple Choice answer sheet provided.
- Allow about 8 minutes for this section.

Section II: **80 Marks**

- Attempt Question 6 - 9
- Answer on paper provided unless otherwise instructed. Start a new page for each new question.
- Allow about 1 hours & 52 minutes for this section.

The answers to all questions are to be returned in separate *stapled* bundles clearly labelled Question 6, Question 7, etc. Each question must show your Candidate Number.

Candidate No:__________

(Tear this page off and hand in)

Yr 12 Maths Exam   Term 1, 2014

Multiple Choice Answer Sheet :

Q1	
Q2	
Q3	
Q4	
Q5	

Total:_______/5

Year 12 Mathematics Assessment 2 2014

Section I Multiple Choice (5 marks)

1. The diagonals bisect the vertex angle through which they pass in the following quadrilateral(s):
   i. Parallelogram ii. Rectangle iii. Square iv. Rhombus

   A. iii only B iii, iv C ii, iii, iv D all of the above

2. The x-ordinate of the point on the curve $y = x^2 + 2x + 3$ at which the tangent is perpendicular to the line $x + 3y + 3 = 0$ is

   A. $-\frac{5}{2}$ B $-\frac{1}{2}$ C $\frac{1}{2}$ D $\frac{5}{2}$

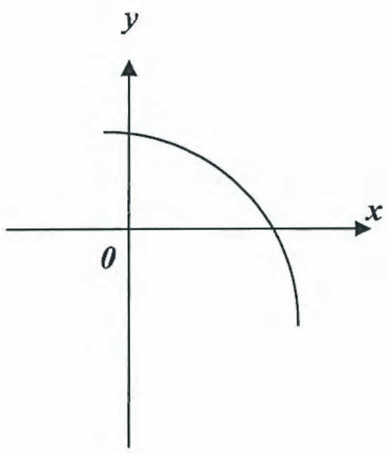
3. If $y = \frac{e^{-x} - e^{x}}{e^{-x} + e^{x}}$, then $\frac{dy}{dx} = ?$

   A. 1 B $\frac{1}{e^{-2x} + e^{2x}}$ C $\frac{4}{(e^{-x} + e^{x})^2}$ D $\frac{-4}{(e^{-x} + e^{x})^2}$

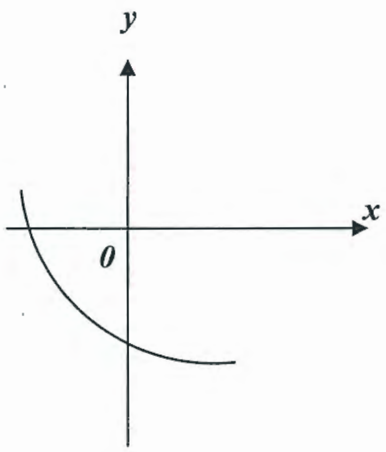
4. The function $f(x) = x^4 - 4x^2$ has
   A. One relative minimum and one horizontal point of inflexion.
   B. One relative minimum and two horizontal points of inflexion.
   C. Two relative minima and one relative maximum.
   D. One relative minimum and two relative maxima.

5. For which curve shown below are $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ both positive?

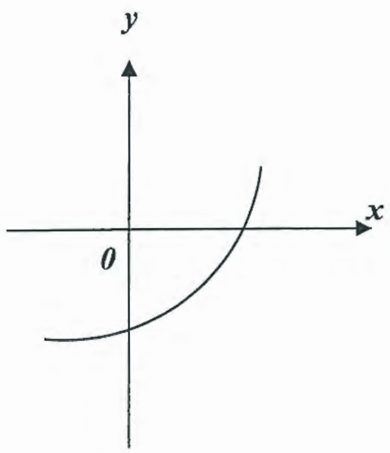
A



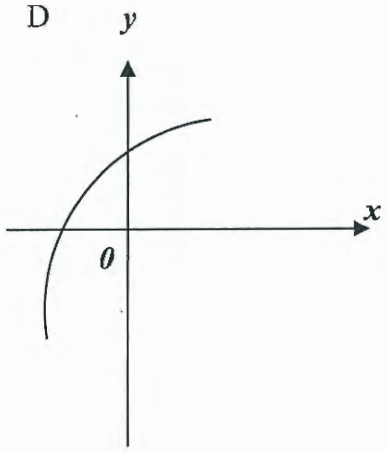
B



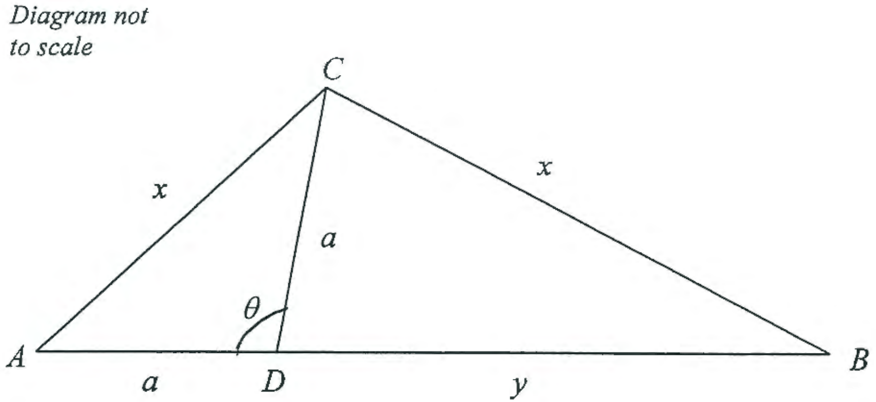
C



D



d. In the diagram below, ABC is an isosceles triangle with $AC=BC=x$. The point D on the interval AB is chosen so that $AD=CD$. Let $AD=a$, $DB=y$ and $\angle ADC=\theta$.



i) Show that ΔABC is similar to ΔACD. 3

ii) Show that $x^2=a^2+ay$. 2

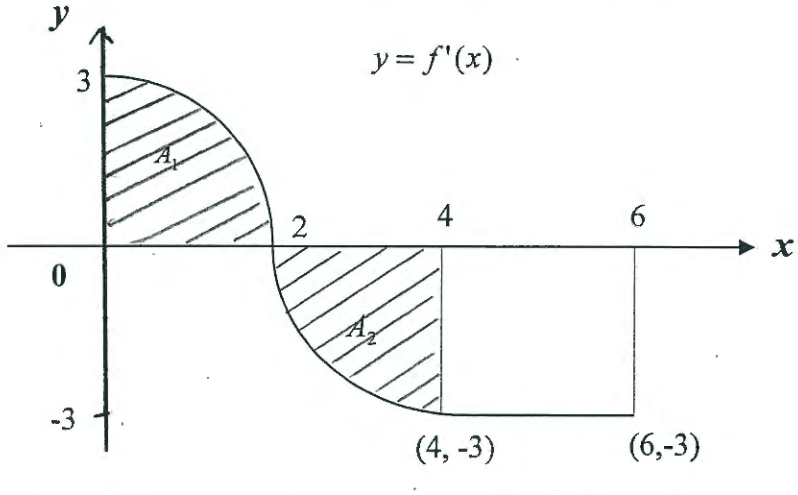
iii) Show that $y=a(1-2\cos\theta)$. 2

iv) Deduce that $y\leq 3a$. 1

Question 8 (20 marks)

a. Simplify $\sum_{k=1}^{n}2(3^k)$. 2

b. Let $y=f(x)$ be a function defined for $0\leq x\leq 6$, with $f(0)=0$. The diagram shows the graph of the derivative of $f(x)$, i.e. $y=f'(x)$.



The shaded regions A_1 and A_2 both have area of 4 square units.

i) For which values of x is $f(x)$ increasing? 1

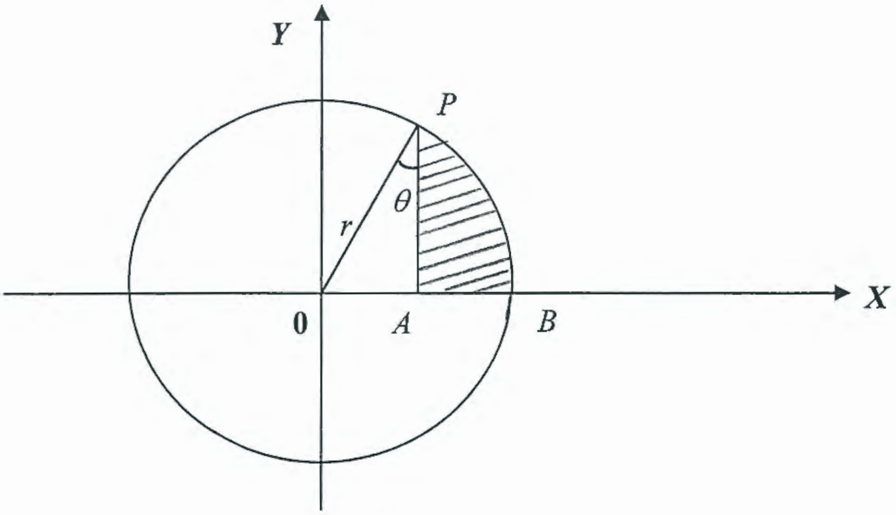
ii) For what value of x does the maximum value of $f(x)$ occur? Give reasons. 1

iii) Show the maximum value of $f(x)=4$. (Hint: Consider $\int_0^2 f'(x)dx$) 1

iv) Find the value of $f(6)$. 2

v) Draw a graph of $y=f(x)$ for $0\leq x\leq 6$. 1

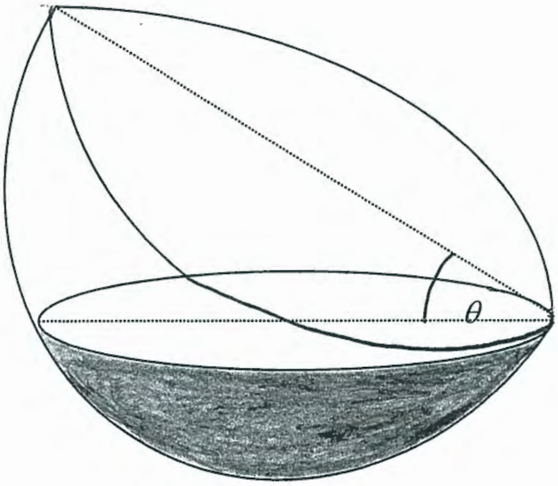
c. The circle $x^2 + y^2 = r^2$ has radius r and centre O. The circle meets the positive x-axis at B. The point A is on the interval OB.
A vertical line through A meets the circle at P. Let $\theta = \angle OPA$.



i) Verify that a point P on the circle can be represented by $(r\sin\theta, r\cos\theta)$. 1

ii) The shaded region bounded by the arc PB and the intervals AB and AP is rotated about the x-axis. Show that the volume, V formed is given by : 3

$$V = \frac{\pi r^3}{3}(2 - 3\sin\theta + \sin^3\theta)$$



iii) A container is in the shape of a hemisphere of radius r metres. The container is initially horizontal and full of water. The container is then tilted at an angle of θ to the horizontal so that some water spills out.

α) Show that the depth of water when $\Theta = \frac{\pi}{6}$ is one half of the original depth. 1

β) What fraction of the original volume is left in the container? 2

End

Section I M.C. Yr 12 [illegible] P.1

1. Ⓑ

2. slope of $x+3y+3=0$ is $-\frac{1}{3}$, $m^{\perp}=3$

$y' = 2x+2 = 3$

$x=\frac{1}{2}$

Ⓒ

3. $y = \frac{e^{-x}-e^{x}}{e^{-x}+e^{x}}$

$$y' = \frac{(e^{-x}+e^{x})[-e^{-x}-e^{x})-(e^{-x}-e^{x})(-e^{-x}+e^{x})}{(e^{-x}+e^{x})^2}$$

$$= \frac{-e^{-2x}-\overset{1}{\cancel{e^{0}}}-\overset{1}{\cancel{e^{0}}}-e^{2x}-[-e^{-2x}+\overset{1}{\cancel{e^{0}}}+\overset{1}{e^{0}}-e^{2x}]}{(e^{-x}+e^{x})^2}$$

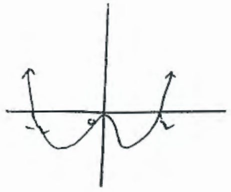
$$= \frac{-\cancel{e^{-2x}}-2-\cancel{e^{2x}}+\cancel{e^{-2x}}-2+\cancel{e^{2x}}}{(e^{-x}+e^{x})^2}$$

$$y' = \frac{-4}{(e^{-x}+e^{x})^2}$$

Ⓓ

4. $y=x^2(x^2-4)$

Ⓒ



5. Ⓒ

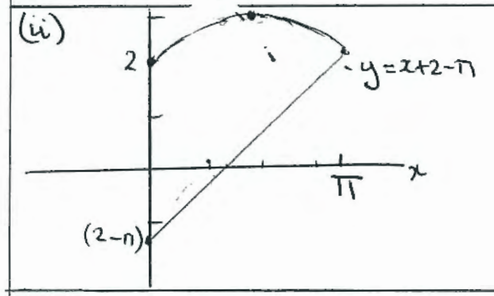
MATHEMATICS: Question...6..

Suggested Solutions	Marks	Marker's Comments
a) $\lim_{x\to 0} \frac{\sin x}{x} = 1$	1	well done
b) (i) $\int (3x-4)^8\,dx$ $= \frac{(3x-4)^9}{3\times 9} + C$ $= \frac{3x-4}{27} + C$	1	must have $+C$.
(ii) $\int \frac{2}{e^x}\,dx = \int 2e^{-x}$ $= -2e^{-x}(+C)$	1	
(iii) $\int \frac{(\sqrt{x}-3)^2}{2\sqrt{x}}\,dx = \int \frac{x-6\sqrt{x}+9}{2\sqrt{x}}\,dx$ $= \frac{1}{2}\int x^{1/2} - 6 + 9x^{-1/2}\,dx$ $= \frac{1}{2}\left[\frac{2}{3}x^{3/2} - 6x + 18x^{1/2}\right]$ $= \frac{1}{3}x^{3/2} - 3x + 9x^{1/2} + C$	1 1 1 ③	some poor algebraic manipulation

MATHEMATICS: Question 6

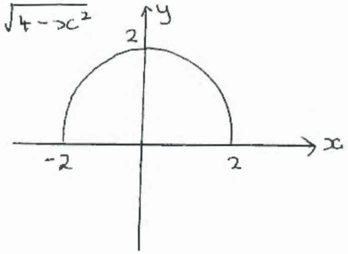
<table>
<tr><th>Suggested Solutions</th><th>Marks</th><th>Marker's Comments</th></tr>
<tr><td>(iv) $\int \frac{x}{16-x^2}\,dx = -\frac{1}{2}\ln(16-x^2)+c$</td><td>2</td><td>1 for ln
1 for the $-\frac{1}{2}$.</td></tr>
<tr><td>(c) (i)
<table>
<tr><td>x</td><td>1</td><td>1.5</td><td>2</td><td>2.5</td><td>3</td></tr>
<tr><td>$y=\frac{1}{x}$</td><td>1</td><td>$\frac{2}{3}$</td><td>$\frac{1}{2}$</td><td>$\frac{2}{5}$</td><td>$\frac{1}{3}$</td></tr>
<tr><td>weights</td><td>1</td><td>4</td><td>2</td><td>4</td><td>1</td></tr>
</table>
$\therefore \frac{\frac{1}{2}}{3}\left[1\times1+4\times\frac{2}{3}+2\times\frac{1}{2}+4\times\frac{2}{5}+1\times\frac{1}{3}\right]$
$= \frac{1}{6}\left[1+\frac{8}{3}+1+\frac{8}{5}+\frac{1}{3}\right]$
$= \frac{11}{10}$</td><td>①
①</td><td>There is no excuse for incorrect calculation calculators need to be used.</td></tr>
<tr><td>(ii) $A=\int_1^3 \frac{1}{x}\,dx$
$= [\ln x]_1^3$
$= \ln 3 - \ln 1$
$= \ln 3$</td><td>1</td><td>well done.</td></tr>
<tr><td>$\ln 3 = \frac{11}{10}$
$e^{\frac{11}{10}} = 3$
$\therefore e = 3^{10/11}$
$= \underline{2.7}$</td><td>1
1
1</td><td>Had to show this line</td></tr>
</table>

MATHEMATICS: Question 6

<table>
<tr><th>Suggested Solutions</th><th>Marks</th><th>Marker's Comments</th></tr>
<tr><td>d(i) $y = 2+\sin x$
$\frac{dy}{dx} = \cos x$
at $x=\pi$ $\frac{dy}{dx} = -1$
$x=\pi$ $y = 2+\sin\pi$
$= 2$.
$M_{normal} = 1$
$\therefore y-2 = 1(x-\pi)$
$\therefore y = x+2-\pi$</td><td>1
1</td><td>As it was a show question had to clearly indicate last step.</td></tr>
<tr><td>(ii)  </td><td>②</td><td>1 for each graph</td></tr>
<tr><td>(iii) $A=\int_0^{\pi} 2+\sin x-(x+2-\pi)\,dx$
$= \int_0^{\pi} \sin x - x + \pi\,dx$
$= \left[\pi x - \frac{x^2}{2} - \cos x\right]_0^{\pi}$
$= \left[\pi^2 - \frac{\pi^2}{2} - \cos\pi\right] - [0-0+\cos 0]$
$A = \frac{\pi^2}{2} + 2u^2$.</td><td>1
1
1</td><td>Students who tried to break regions up were less successful than those who subtracted regions</td></tr>
</table>

Question 7

a i) $y = \sqrt{4 - x^2}$



✓ 1 Mark.

check - not a parabola

ii) $\int_{-2}^{2} \sqrt{4 - x^2}\, dx = \frac{1}{2}\pi r^2$

$= \frac{1}{2} \times \pi \times 2^2$

$= 2\pi$

✓ 1 Mark.

$\int_{-3}^{2}$ - find area between $x = 2$ and $x = -2$.

b i)

Total $= 2(100) + 2(200) + 2(300) \ldots\ldots 2(2000)$

$= 2(100 + 200 + 300 + 400 \ldots\ldots 2000)$

$S(n) = 2\left[\frac{n}{2}\left(2a + (n-1)d\right)\right]$

$n = 20$
$d = 100$
$a = 100$

$= 2\left[\frac{20}{2}\left(2 \times 100 + (20-1)100\right)\right]$

$= 2\left[10\left(200 + 19 \times 100\right)\right]$

$= 2\left[21000\right]$

$= 42000$ m

∴ Truck travels 42 km.

✓ Correct substitution
✓ answer

Some got 40,000 m ie 40km

They used $\frac{n}{2}\left(a + (n-1)d\right)$ instead of $\frac{n}{2}\left(2a + (n-1)d\right)$

ii) $S_n = 21000$ - half the distance

$2[100 + 200 + 300 \ldots\ldots 2000) = 21000$

$= [100 + 200 + 300 \ldots\ldots n(100)] = 10500$

$S_n = \frac{n}{2}(a + l)$

$10500 = \frac{n}{2}(100 + 100n)$

$21000 = n(100 + 100n) - 21000$

$0 = 100n^2 + 100n$

$0 = n^2 + n - 210$

$0 = (n+15)(n-14)$

$n = -15$ or $n = 14$

∴ $n = 14$ (as $n > 0$)

ie After 14 trips, the truck has travelled half of the total distance

✓ correct ~~substition~~ substitution
✓ answer with conclusion

you can also use $\frac{n}{2}\left[200 + (n-1)d\right]$

$200 + 400 + 600 \ldots\ldots 4000$

$S_n = \frac{n}{2}\left(2(200) + (n-1)200\right)$

$= \frac{n}{2}\left(400 + 200n - 200\right)$

$= \frac{n}{2}\left(200 + 200n\right)$

$21000 = 100n + 100n^2$

$0 = 100n^2 + 100n - 21000$

$0 = n^2 + n - 210$

$0 = (n+15)(n-14)$

$n = -15$ or $n = 14$

4c(i) LHS $= \frac{x-2}{x+3}$

$$x+3 \overline{)\, x-2 \,}^{\,1}$$

$x + 3$
-5

∴ $\frac{x-2}{x+3} = 1 - \frac{5}{x+3}$

OR

$\frac{x-2}{x+3} = \frac{x+3-5}{x+3}$

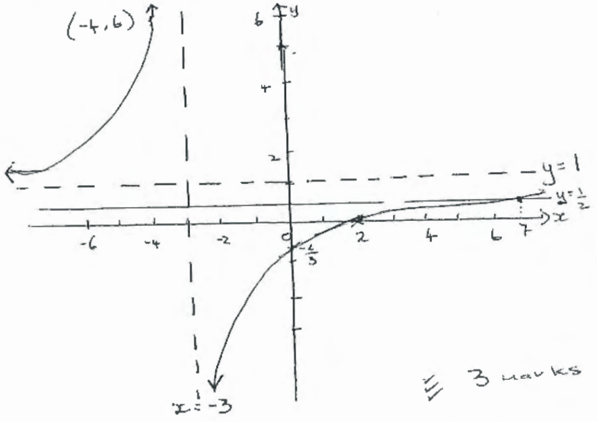
$= \frac{x+3}{x+3} - \frac{5}{x+3}$

$= 1 - \frac{5}{x+3}$

✓ 1 Mark

Very well done

c(ii).



✓✓✓ 3 marks

Marks awarded for:

✓ Asymptotes → $x = -3$
→ $y = 1$

✓ shape / intercepts

Becareful curve does not appear to cut the asymptotes (lost one mark)

✓ position correct.

(iii) $\frac{x-2}{x+3} > \frac{1}{2}$

From graph: $x > 7$ and $x < -3$

algebraically: $\frac{x-2}{x+3} = \frac{1}{2}$

$2x - 4 = x + 3$

$x = 7$

∴ $x > 7$ and $x < -3$

✓ getting 7 and -3
✓ correct inequality.

d(i) △ABC ||| △ACD

Method 1 - Prove equiangular

In triangle ACD, AD = CD = a (given)

∴ ∠CAD = ∠ACD (angles opposite equal sides are equal)

Similarly in triangle ABC

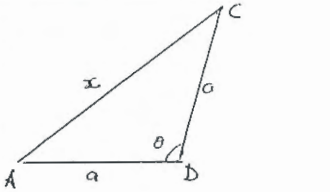
∠CAB = ∠CBA

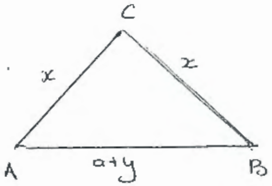
∴ ∠CAB = ∠CAD (same angle.

∠CAD = ∠ACD = ∠CAB = ∠CBA

∴ △ABC ||| △ACD (equiangular)

Note (If using the reason that base angles of Isoceles triangle are equal" you have to mention which sides in the isoceles triangle are equal)





In ΔADC

$A\hat{D}C = \theta$ (data)

$\hat{A} = \hat{C}$ (equal angles are opposite equal sides in ΔADC)

$2\times\hat{A} + \theta = 180$ (angle sum of ΔACD is 180°)

$\hat{A} = \frac{180-\theta}{2}$

$\hat{A} = 90 - \frac{\theta}{2}$

$\hat{A} = \hat{C} = 90^\circ - \frac{\theta}{2}$

in ΔABC

$\hat{A} = \hat{B}$ (equal angles are opposite equal sides in ΔABC)

$\hat{A} = 90 - \frac{\theta}{2}$

$\hat{B} = 90 - \frac{\theta}{2}$

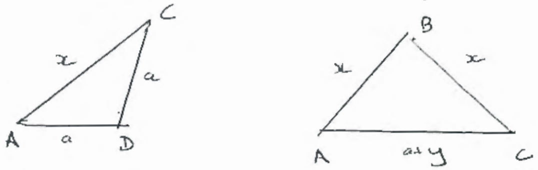
$\therefore$ In Δ's ABC and ACD

$\hat{A}$ is common

$\hat{C} = \hat{B}$ (both $90 - \frac{\theta}{2}$)

$\therefore \Delta ABC ||| \Delta ACD$ (equiangular)

Method 3 Ratio of corresponding sides



For $\Delta ABC ||| \Delta ACD$

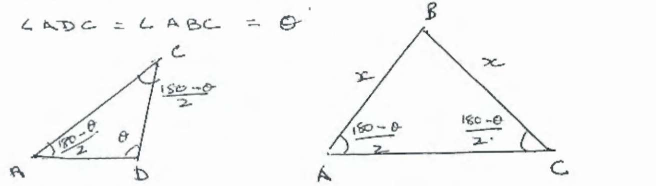
$\frac{BC}{CD} = \frac{x}{a}$

and $\frac{AC}{AD} = \frac{x}{a}$

$\therefore \frac{BC}{CD} = \frac{AC}{AD}$ (same ratio of corresponding sides in similar triangles

To prove the angle, we need to prove

$\angle ADC = \angle ABC = \theta$



$\therefore \overset{ABC}{\cancel{ACB}} = 180 - 2\left(\frac{180-\theta}{2}\right)$

$= \theta$

$\therefore$ proved $\angle ADC = \angle ABC = \theta$

2 marks — reasoning, proof

Note cannot prove with SAS as it is not a congruent triangle

Most said angle A is common but one has to mention the included angle.

d(ii) $\frac{CA}{AB} = \frac{AD}{AC}$ (corresponding sides of similar triangles are in the same ratio $\Delta ABC ||| \Delta ACD$

$\frac{x}{a+y} = \frac{a}{x}$

$x^2 = a(a+y)$

$x^2 = a^2 + ay$.

Most did well

✓ ratio of sides

✓ getting equation

(iii) In triangle ACD, using cosine Rule.

$x^2 = a^2 + a^2 - 2a^2 \cos\theta$.

$a^2 + ay = 2a^2 - 2a^2\cos\theta$ (from part (ii)

$ay = a^2 - 2a^2\cos\theta$.

$ay = a^2(1 - 2\cos\theta)$

$\therefore y = a(1 - 2\cos\theta)$

✓ correctly substituting $x^2 = a^2 + ay$ into cosine formula

✓ steps leading to answer

iv) $2\cos\theta$ lies between -2 and 2

when $2\cos\theta = 2 \Rightarrow y = a(1-2)$

$y = -a$

when $2\cos\theta = -2 \Rightarrow y = a(1+2)$

$y = 3a$.

$\therefore -a \le y \le 3a$

ie. $y \le 3a$. ✓ 1 mark.

recognising that Cos graph is between 1 and -1 is worth a mark

- Most students did not attempt this question.

Term 1 Task 2 2U Year 12 2014

(a) $\sum_{k=1}^{n} 2(3^k) = 2(3^1+3^2+3^3+\dots+3^n)$

$= 2\left[\frac{3(3^n-1)}{3-1}\right]$ ①

$= 3(3^n-1)$ ①

2 marks

(b)(i) For increasing function $f'(x)>0$

$\therefore 0 \leq x < 2$ ①

$\frac{1}{2}$ mark was paid for $0 \leq x \leq 2$ or $0<x<2$

1 mark

(ii) Max. value of $f(x)$ occurs at $x=2$ as $f'(x)>0$ for $0 \leq x < 2$ and $f'(x)<0$ for $x>2$.

No marks if there was not a satisfactory reason and only $x=2$ was given.

1 mark

(iii) Given $\int_0^2 f'(x)\,dx$ and $A_1=A_2=4$

$\int_0^2 f'(x)\,dx = \left[f(x)\right]_0^2$

$= f(2)-f(0)$

* Reasons were needed for the 1 mark

Now $4 = f(2)$ as $f(0)=0$ (given)

$\therefore$ Max. value of $f(x)=4$ when $x=2$

1 mark

(iv) $\int_2^6 f'(x)\,dx = \left[f(x)\right]_2^6$

$= f(6)-f(2)$

$= f(6)-4$ from (iii) ①

Now $\int_2^6 f'(x)\,dx = \int_2^4 f'(x)\,dx$ + Area of Rectangle

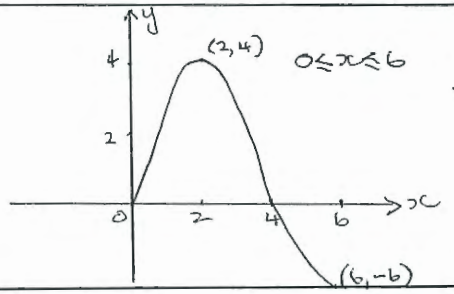
$= -4 + (-2\times3)$ (both areas are under the curve)

$= -10$

$\therefore -10 = f(6)-4$

$\therefore \underline{f(6) = -6}$ ①

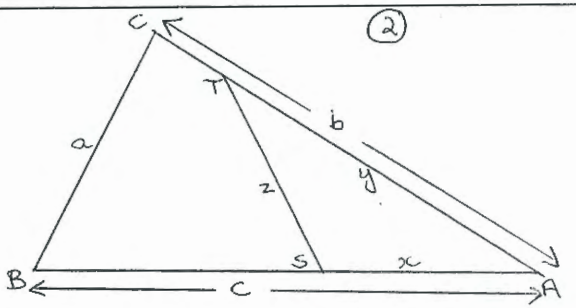
(v)



$0 \leq x \leq 6$ Needed the domain to score a mark

1 mark

(c)



(i) Area $\triangle ABC$ = 2 Area $\triangle SAT$ ① as TS divided the area $\triangle ABC$ in half.

$\frac{1}{2}bc\sin A = 2\times\frac{1}{2}xy\sin A$ ①

$\frac{1}{2}bc = xy$

$\therefore xy = \frac{1}{2}bc$

2 marks

(ii) In $\triangle AST$

$z^2 = x^2+y^2-2xy\cos A$ ①

$z^2 = x^2+y^2-2\times\frac{1}{2}bc\cos A$ as $xy=\frac{1}{2}bc$ from (i)

$z^2 = x^2+\left(\frac{\frac{1}{2}bc}{x}\right)^2 - bc\cos A$ ①

$\therefore z^2 = x^2+\frac{b^2c^2}{4x^2}-bc\cos A$

2 marks

(iii) $\frac{d(z^2)}{dx} = 2x - 2\left(\frac{b^2c^2}{4x^3}\right)$

$= 2x - \frac{b^2c^2}{2x^3}$ ①

For stationary points $\frac{d(z^2)}{dx}=0$

$\therefore 0 = 2x - \frac{b^2c^2}{2x^3}$ ①

$\frac{2x(2x^3)-b^2c^2}{2x^3}=0$

$4x^4-b^2c^2=0$

$4x^4 = b^2c^2$

$x^4 = \frac{b^2c^2}{4}$

$\therefore x = \sqrt[4]{\frac{b^2c^2}{4}}$ ①

$x = \sqrt{\frac{bc}{2}}$ as $\left(\sqrt{\frac{bc}{2}}\times\sqrt{\frac{bc}{2}}\times\sqrt{\frac{bc}{2}}\times\sqrt{\frac{bc}{2}}\right)^{\frac{1}{4}} = \sqrt[4]{\frac{b^2c^2}{4}}$

and $x>0$

Test nature of the stationary point

$\frac{d^2(z^2)}{dx^2} = \frac{d}{dx}\left[2x - \frac{b^2c^2}{2}x^{-3}\right]$

$= 2 + \frac{3b^2c^2}{2x^4}$

$= 2 + \frac{3}{2}(b^2c^2)\times\frac{1}{\left(\frac{b^2c^2}{4}\right)}$ as $x^4=\frac{b^2c^2}{4}$

$= 2 + \frac{3}{2}(\cancel{b^2c^2})\times\frac{4}{(\cancel{b^2c^2})}$

$$\therefore \frac{d^2(z^2)}{dx^2} = 2 + \frac{3}{2} \times 4$$

$$= 8$$

① for the test.

As $\frac{d^2(z^2)}{dx^2} > 0$ the curve is concave up and hence a relative minimum occurs at $x = \sqrt{\frac{bc}{2}}$. As there is only one turning point for $x > 0$ the local minimum is also the absolute minimum.

4 marks

(iv) In $\triangle ABC$

$$\cos A = \frac{b^2+c^2-a^2}{2bc}$$

① mark

(v)

$$z^2 = x^2 + \frac{b^2c^2}{4x^2} - bc\cos A$$

from (ii)

$$= \frac{bc}{2} + \frac{b^2c^2}{4\frac{bc}{2}} - bc\cos A$$

when $x = \sqrt{\frac{bc}{2}}$

$x^2 = \frac{bc}{2}$

$$= \frac{bc}{2} + \frac{bc}{2} - bc\cos A$$

$$= bc - bc\cos A \quad ①$$

$$= bc(1 - \cos A)$$

$$= bc\left(1 - \frac{(b^2+c^2-a^2)}{2bc}\right)$$

from (iv)

$$= bc\left(\frac{2bc-b^2-c^2+a^2}{2bc}\right) \quad ①$$

$$= \frac{2bc-b^2-c^2+a^2}{2} \quad ①$$

$$z^2 = \frac{(a^2) - (b-c)^2}{2}$$

$$\therefore z = \sqrt{(a-b+c)(a+b-c)}$$

3 marks

Qu 8.

Very badly attempted by most students. Marks ranged from $\frac{0}{20}$ (1 student) to $\frac{20}{20}$ (2 students)

Student's algebra was poor and their lack of detail in their working was disappointing.

Part (a) was reasonably well done.

Part (b) (iii) and (iv) was particularly poor as students did not appear to understand how to interpret the $f'(x)$ graph and find a value for $f(6)$.

Part (c) was badly done as students could not see the link between the areas in Qu(i) — this is in the enrichment course so favoured the enrichment students. (c)(ii) was reasonably done.

(c)(iii) was poorly done as students tried to find the derivative of a surd. Testing the nature was poorly done.

(c)(iv) an easy question but students thought it was harder than it actually was so received no marks.

(c)(v) If (c)(iv) was incorrect they did not get the question out. Algebra was poor.

ANSWERS TO QUESTION 9

(a) (i) $\frac{d}{dx}\left(\frac{lnx}{x}\right)$

$$= \frac{x \times \frac{1}{x} - (lnx) \times 1}{x^2}$$

$$= \frac{1}{x^2} - \frac{lnx}{x^2}$$

[To obtain the mark students needed to show that they have applied either the quotient rule or the product rule:

This could be done either by including the second line of working

OR

by showing the U, U′, V, V′ components AND writing down the rule.

Writing the components and then the answer was not sufficient for a "show that" mark.]

(a) (ii) The numbering of the question *(i)* and then *(ii)* gave a clear indication the parts were linked.

The "hence" meant that (i) needed to be used to answer (ii).

$$\frac{d}{dx}\left(\frac{lnx}{x}\right) = \frac{1}{x^2} - \frac{lnx}{x^2}$$

The approach to use here is to make $\frac{lnx}{x^2}$ the subject and then to integrate each term with respect to x.

$$\frac{lnx}{x^2} = \frac{1}{x^2} - \frac{d}{dx}\left(\frac{lnx}{x}\right)$$

$$\int_1^e \frac{lnx}{x^2} dx = \int_1^e \frac{1}{x^2} dx - \int_1^e \frac{d}{dx}\left(\frac{lnx}{x}\right) \mathrm{dx}$$

$$\int_1^e \frac{lnx}{x^2} dx = \left[\frac{-1}{x}\right]_{x=1}^{x=e} - \left[\frac{lnx}{x}\right]_{x=1}^{x=e}$$

$$= \frac{-1}{e} - \frac{-1}{1} - \left[\frac{lne}{e} - \frac{ln1}{1}\right]$$

$$= 1 - \frac{2}{e}$$

3 marks for correct answer

2 marks for one computational error or failure to simplify *lne*

2 marks for all correct, but failure to substitute in bounds of integration for $\frac{lnx}{x}$

1 mark for correct use of (i)

COMMENT:

It was alarming to see students writing

$$\int_1^e \frac{1}{x^2}$$

with no variable of integration. This is bad mathematics. You must always integrate with respect to something!

(b) (i) $r = 0 \cdot 005$ *(per month)*

$1 + r = 1 \cdot 005$

The pattern must be built up by listing (at least) 3 iterations, before introducing the sum of a GP formula. This is a *show that* question (not a quote the formula problem!)

$A_1 = 300000(1 \cdot 005) - 2000$

$A_2 = [300000(1 \cdot 005) - 2000] \times 1 \cdot 005] - 2000$

$A_2 = 300000(1 \cdot 005)^2 - 2000(1 + 1 \cdot 005)$

$A_3 = ([300000(1 \cdot 005) - 2000] \times 1 \cdot 005 -$

$2000) \times 1 \cdot 005 - 2000$

$A_3 = 300000(1 \cdot 005)^3 - 2000(1 + 1 \cdot 005 + 1 \cdot 005^2)$

$A_k = 300000(1 \cdot 005)^k - 2000(1 + 1 \cdot 005 +$

$1 \cdot 005^2 + \ldots + 1 \cdot 005^{k-1})$

$A_k = 300000(1 \cdot 005)^k - 2000\left(\frac{1 \cdot 005^k - 1}{1 \cdot 005 - 1}\right)$

$A_k = 300000(1 \cdot 005)^k - 400000(1 \cdot 005^k - 1)$

$A_k = 400000 - 100000(1 \cdot 005^k)$

$A_k = 100000(4 - 1 \cdot 005^k)$

(b) (ii) $k = 9 \times 12 = 108$

$A_{108} = 100000(4 - 1 \cdot 005^{108})$

$= 228630 \cdot 0501 \ldots$

$= \$228\ 630$ *(nearest dollar)*

After 9 years

\$228 630 *is the balance of the loan*

(b) (iii) When the loan is paid off, $A_n = 0$

$100000(4 - 1 \cdot 005^n) = 0$

$4 = 1 \cdot 005^n$

$n = \frac{ln4}{ln1 \cdot 005} = 277 \cdot 95 \rightarrow 278$ *full months*

(b) (iv) Originally the loan would have taken 278 months to pay off. Before the financial crisis, there would have been $12 \times 9 = 108$ *months*. A further 18 months passed during the 'repayment free period'. This means that 278 – (108+18) = 152 months would be available to pay off the loan.

During the repayment free period, additional interest accrued.

New balance = $228630 \cdot 0501(1 \cdot 005)^{18}$

= \$250105.0283

=\$250105 (nearest dollar)

Let $\$M$ be the new payment

$250105.03 \times 1.005^{152} = M\left(\frac{1 \cdot 005^{152} - 1}{1 \cdot 005 - 1}\right)$

$M = \frac{250105.03(0 \cdot 005) \times 1.005^{152}}{1 \cdot 005^{152} - 1}$

M = \$2353 (nearest dollar)

(c) (i) Either . . .

$x^2 + y^2$

$= (rsin\theta)^2 + (rcos\theta)^2$

$= r^2(sin^2\theta + cos^2\theta)$

$= r^2$

$\therefore$ *P is on the circle because it satisfies the equati*

$x^2 + y^2 = r^2$

OR

Use right-angled triangles to show clearly that the x and y coordinates can be expressed as $rsin\theta$ and $rcos\theta$ respectively

(c) (ii)

$$V = \int_{rsin\theta}^{r} \pi y^2 dx$$

$$V = \pi \int_{rsin\theta}^{r} (r^2 - x^2)dx$$

$$V = \pi[r^2x - \frac{x^3}{3}]_{x = rsin\theta}^{x = r}$$

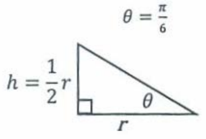
$$V = \pi[\left(r^2(r) - \frac{(r)^3}{3}\right) - \left(r^2(rsin\theta) - \frac{(rsin\theta)^3}{3}\right)]$$

$$V = \pi[\frac{2}{3}r^3 - r^3sin\theta + \frac{r^3sin^3\theta}{3}]$$

$$V = \frac{\pi r^3}{3}[2 - 3sin\theta + sin^3\theta]$$

Rotate the diagram at the top of the page 90° clockwise. Produce the radius to form a diameter.

(d) (α) $sin\theta = \frac{1}{2}$



(β) when $\theta = \frac{\pi}{6}$

$$V = \frac{\pi r^3}{3}[2 - 3sin\frac{\pi}{6} + sin^3\frac{\pi}{6}]$$

$$V = \frac{\pi r^3}{3}[2 - \frac{3}{2} + (\frac{1}{2})^3]$$

$$V = \frac{\pi r^3}{3}[2 - \frac{3}{2} + \frac{1}{8}]$$

$$= \frac{5\pi r^3}{24}$$

Original volume is when $\theta = 0$.

That is,

$$V = \frac{\pi r^3}{3}[2 - 3sin0 + sin^3 0]$$

$$V = \frac{2\pi r^3}{3}$$

$$\frac{New\ volume}{Original\ volume} = \frac{\frac{5\pi r^3}{24}}{\frac{2\pi r^3}{3}}$$

$$= \frac{5}{16}$$